Two-stage uncapacitated facility location, set packing and forests

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Abstract

The two–stage uncapacitated facility location problem (TUFLP), in which a product is transported from the production plants to certain depots or distribution centers and then to the customers, is considered. The location of plants and depots forms part of the decision problem, and it takes into account the installation costs of the different plants and depots and the transportation costs associated with both stages. It is assumed that the production capacity of each plant and the distribution capacity of each depot are unlimited.

In order to address the problem by means of integer programming techniques, two main families of formulations have been considered. The formulations used in this work is based on three times–indexed variables, covering the transportation of the product from the production plant to the final destination, but it is assumed that the transportation costs for each of the stages are separately available.

The interest here is on the polyhedral structure of the convex hull of the feasible solutions of the problem. Concretely, new facet-defining inequalities for the polyhedron associated with an enforced formulation are obtained. When these inequalities are added to the linear relaxation of TUFLP (where the integrality constraints are removed), fractional solutions of this relaxation are cut off, and the performance of the branch-and-cut methods hopefully improves.

The model contains the following elements. A set $K = \{1, \ldots, q\}$ of potential plants with installation costs $g_k > 0$, $k \in K$; a set $J = \{1, \ldots, m\}$ of potential depots with installation costs $f_j > 0$, $j \in J$; a set $I = \{1, \ldots, n\}$ of customers demanding $D_i > 0$ units of a product, $i \in I$; costs $c_{ij} > 0$ of transportation of one unit of product from depot j to customer i, $i \in I$, $j \in J$; and costs $d_{kj} > 0$ of transportation of one unit of product from plant k to depot j, $k \in K$, $j \in J$.

The aim is to satisfy the demand of all the customers, minimizing the total cost of the system. This cost is obtained by summing up the transportation costs of both stages, the installation costs of all the depots which send product to any customer, and the installation costs of all the plants which send product to any depot.

The problem can be formulated by using one set of binary transportation variables x_{ijk}, $i \in I$, $j \in J$, $k \in K$, taking the value one if the demand of i is supplied through k and j; binary variables y_j, $j \in J$, taking the value 0 if depot j is opened; binary variables z_k, $k \in K$, taking the value 0 if plant k is opened; and a huge number nM:

$$
\begin{aligned}
\text{(SPTUFLP) Max } & \sum_{j=1}^{m} f_j y_j + \sum_{k=1}^{q} g_k z_k + \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{q} C'_{ijk} x_{ijk} \\
\text{s.t. } & \sum_{j=1}^{m}\sum_{k=1}^{q} x_{ijk} \leq 1, \qquad \forall i \\
& x_{ijk} + y_j \leq 1, \quad \forall i\ \forall j\ \forall k \\
& x_{ijk} + z_k \leq 1, \quad \forall i\ \forall j\ \forall k \\
& y_j \in \{0,1\}, \qquad \forall j \\
& z_k \in \{0,1\}, \qquad \forall k \\
& x_{ijk} \in \{0,1\}, \quad \forall i\ \forall j\ \forall k,
\end{aligned}
$$

where $C'_{ijk} = M - C_{ijk} > 0\ \forall i, \forall j, \forall k$. This is a set packing problem.

By noting that (SPTUFLP) always has an optimal solution where $x_{ijk} = 1 \Rightarrow x_{i'jk'} = 0$ for all i, i', j and all $k \neq k'$, new constraints

$$x_{ijk} + x_{i'jk'} \leq 1 \qquad \forall j, i, i', k \neq k'$$

can be used to break the symmetry of the original formulation and enforce it.

Cliques, odd holes, fans, grilles, and other structures which are present in the intersection graph associated with the enforced formulation have been used to obtain many new families of new valid inequalitites which are facets of the convex hull of the set of feasible solutions of the original formulation that also satisfy the new constraints. Some of these new families can be easily separated –in polynomial time– and used to design a branch and cut method to solve the problem. The implementation of this method is the matter of our present work.

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